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Natural convection in water-saturated reticulated vitreous carbon foam

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ABSTRACT

Experiments of the Horton–Rogers–Lapwood type are reported for water-saturated, 97% porous reticulated vitreous carbon foam. Heat transfer data have been obtained for layer aspect ratios of 2.5 and 5, porous medium Rayleigh numbers from 140 to 4700, and Darcy numbers from 5×10^{-5} to 37×10^{-5}. The data are well correlated in the form

$$Nu_m = (0.007 \pm 0.002)Ra_m^{0.504 \pm 0.06}\ Pr_p^{0.41 \pm 0.08}.$$

Conduction through the solid phase plays a limited role, and the main heat transfer mechanism is advection. Correlation of the present data and water-saturated copper foam data [1] is achieved by inclusion of the conductivity ratio to give,

$$Nu_m = (0.008 \pm 0.003)\left(\frac{k_m}{k_f}\right)^{0.25 \pm 0.04} Ra_m^{0.50 \pm 0.02} Pr_p^{0.38 \pm 0.04}.$$

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1. Introduction

This study expands the work conducted by Kathare et al. [1] on natural convection in water-saturated open cell copper foam to water-saturated reticulated vitreous carbon (RVC) foam. The present experiments are viewed as a step toward a universal correlation for free convection in saturated foam systems. Additionally comparison of the heat transfer data for the copper–water and RVC–water experiments provides empirical insight into the roles of the conductivity ratio, boundary effects, and possibly non-equilibrium phenomena on the heat transfer coefficient. Prior work that forms the general basis for the present study considers natural convection in saturated metal and non-metal foams [1–3], in foam-like structures [4], and in bottom-heated packed beds [5–7]. It should be noted that currently available heat transfer coefficients in free convection for high conductivity foams generally do not follow existing packed bed correlations based on Ra_m. Whether buoyant convection in a saturated foam layer can be represented as that for a packed bed can be conclusively addressed as the range of thermophysical property ratios is expanded.

2. Apparatus and procedure

The system of this study is the rigid wall cylindrical cavity (12.7 cm DIA and variable height) in which unidirectional heat transfer via buoyant convection occurs between two boundaries of known temperature with a known applied heat flux at the lower surface. The RVC layer fills the entire interior of the cavity, and conductive thermal paste assures good thermal contact between the RVC and the upper and lower boundaries. A cross section of the apparatus is shown in Fig. 1, and details of its design, temperature measurements, and heat loss estimates are given by Kathare et al. [1,8].

The foam is RVC of 10 and 20 PPI (pores per inch) with a bulk density of 1.65 gm/cm^3 and a bulk porosity of 97%. The material conductivity of the foam is 6 W/mK and thus provides an excellent contrast to copper foam ($k_s \sim 390$ W/mK). With water as the saturating fluid, $k_s/k_f \sim 10$ compared to the copper–water ratio of $\sim$650. In addition to the solid:fluid conductivity ratio, the effective thermal conductivity, form drag, Brinkman effect, thermal dispersion, and departure from local thermal equilibrium are considered as factors in determining overall heat transfer coefficients.

Characterization of the RVC foam (Table 1) is obtained from direct measurements of the effective stagnant thermal conductivity, permeability, and Forchheimer constant. Experimental uncertainties are computed from zero-level uncertainties and standard procedures to determine propagation of error. The effective stagnant thermal conductivity of $\sim$0.7 W/mK is measured with a stable temperature gradient across the foam–water layer. In comparison, water-saturated open cell copper foam of either 10 or 20 PPI has a stagnant thermal conductivity of $\sim$8.8 W/mK [1]. Hydrodynamic parameters are determined by measurement of pressure drop across a foam sample mounted in a wind tunnel, and permeability and the Forchheimer constant are deduced from these data following the procedures described by Kathare et al. [1,8]. The permeability of the 10 PPI foam is approximately twice as high,

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Nomenclature

A_m	cross sectional area of the porous medium, m^2
C	specific heat, J/kgK
C_F	Forchheimer coefficient
d	diameter, m
D	diameter of test cell, m
Da	Darcy number, K/L^2
E	heat transfer enhancement factor, Eq. (4)
E_{adv}	advective heat transfer enhancement factor, Eq. (5)
g	constant of gravitational acceleration, m/s^2
L	thickness of foam, m
k	thermal conductivity, W/mK
K	permeability, m^2
Nu_f	fluid Nusselt number, $qL/Amkf\Delta T$
Nu_m	porous medium Nusselt number, $(k_f/k_m)Nu_f$
Pr	Prandtl number, $\mu C/k_f$
Pr_m	porous medium Prandtl number, $\mu C/k_m$
Pr_p	effective porous medium Prandtl number, $Pr_m(L/C_fK^{0.5})$
q	net heat transfer rate, W
Ra_f	fluid Rayleigh number, $g\beta\Delta TL3/(\alpha\nu)_f$
Ra_m	porous medium Rayleigh number, $Ra_fDa(k_f/k_m)$
u_m	mean velocity of natural convection, m/s
T	temperature, K
ΔT	temperature difference, T_h-T_c, K

Greek symbols

α	thermal diffusivity, m^2/s
β	isobaric coefficient of thermal expansion, K^{-1}
δ	thickness of thermal boundary layer, m
μ	dynamic viscosity, kg/ms
ν	kinematic viscosity, m^2/s
ρ	density, g/m^3

Subscripts

c	cold
d	dispersion
f	fluid
h	hot
l	ligament
m	porous medium
p	pore
s	solid

$2.4 \pm 0.2 \times 10^{-7}$ m^2, as that of the 20 PPI foam, $1.25 \pm 0.08 \times 10^{-7}$ m^2. Increased pore density has been shown to correlate to decreasing permeability, but pore density is not expected to correlate with the form drag coefficient. Permeability and form drag coefficient for the 20 PPI sample are similar to measurements reported by Bhattacharya et al. [9].

3. Results

3.1. Heat transfer correlations

Heat transfer rates in buoyant convection are measured for $140 < Ra_m < 4700$ and $5 \times 10^{-5} < Da < 37.1 \times 10^{-5}$. In development of the heat transfer correlations, fluid properties are evaluated at the average of the lower and upper surface temperatures. A base case with water only was also measured for comparison [1], and Nusselt numbers are well represented in our apparatus by the Garon–Goldstein [10] correlation for $2.1 \times 10^6 < Ra_f < 2 \times 10^8$. Table 2 contains a summary of the major heat transfer parameters for the present study.

Nusselt numbers are plotted in Fig. 2 versus the porous medium Rayleigh number for specified Darcy number, along with Nusselt numbers reported for copper foam. One of the primary findings of Kathare et al. [1] is the Nusselt numbers for copper foam do not follow the Elder [7] and the Wang–Bejan [11] heat transfer correlations with Rayleigh number for a packed bed of spheres. Nusselt numbers are 27–42% less than that predicted by these correlations for Da ~10^{-5} at sufficiently large Ra_m (>150). In the range of Rayleigh number examined in the present work, Nusselt numbers for RVC foam follow the same trend as for copper foam and an order of magnitude deviation from the packed bed relation is observed at $Ra_m \sim 4700$. Heat transfer associated with the high thermal conductivity of the copper foam is considered to be a possible cause of the observed deviation from the linear Nusselt number relationship [1]. However, the combined data (Fig. 2) suggests that the foam matrix conductivity and solid:fluid conductivity ratio are not the primary cause of deviation from the packed bed relation.

Heat transfer data for water-saturated copper foam [1] were correlated with,

$$Nu_m = (0.007 \pm 0.005)Ra_m^{0.54 \pm 0.08}Pr_p^{0.48 \pm 0.10}, \tag{1}$$

where $44 \leqslant Ra_m \leqslant 216$, $0.37 \leqslant Pr_m \leqslant 0.54$, $1.2 \times 10^{-5} < Da < 2.54 \times 10^{-5}$, and $R^2 = 0.996$. The present RVC data are best correlated with,

$$Nu_m = (0.007 \pm 0.002)Ra_m^{0.50 \pm 0.6}Pr_p^{0.41 \pm 0.08}, \tag{2}$$

where $148 \leqslant Ra_m \leqslant 4700$, $4.4 \leqslant Pr_m \leqslant 6.2$, $4.99 \times 10^{-5} \leqslant Da \leqslant 3.7 \times 10^{-4}$, and $R^2 = 0.993$.

Scale analysis predicts Nusselt number growth with the square root of the Rayleigh number–Prandtl number product in inertial drag dominated flow [11]. This scaling, nearly seen in the correlation for the copper foam, removes the conductivity ratio dependence. Exclusion of the conductivity ratio is thus reasonable when advection dominates owing to the diminished contribution

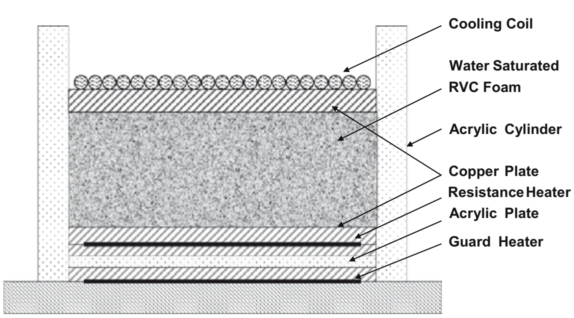

Fig. 1. Cross section of experimental apparatus.

Table 1
Thermal and hydrodynamic parameters for RVC–water layers.

L (mm)	PPI	k_m (W/mK)	$K \times 10^7$ (m^2)	C_F
0.254	10	0.678 ± 0.035	2.4 ± 0.2	0.081 ± 0.003
	20	0.692 ± 0.035	1.25 ± 0.08	0.75 ± 0.04
0.499	10	0.745 ± 0.038	2.4 ± 0.2	0.081 ± 0.003
	20	0.744 ± 0.038	1.25 ± 0.08	0.75 ± 0.04

Table 2
Heat transfer parameters and dimensionless groups.

Foam layer	D/L	k_m (W/mK) (±5%)	$Da \times 10^5$ (±7%)	Ra_f (±4%)	Ra_m (±9%)	Nu_m (±11%)	Pr_m
10 PPI (L = 0.0254 m)	5	0.68	37.1	7.2×10^5–8.2×10^6	230–2710	2.8–9.3	4.9–6.2
20 PPI (L = 0.0254 m)	5	0.69	19.3	8.7×10^5–9.1×10^6	140–1530	2.2–8.6	4.7–5.9
10 PPI (L = 0.0499 m)	2.5	0.74	9.59	1.8×10^7–6.1×10^7	1370–4700	8.6–15.8	4.7–5.5
20 PPI (L = 0.0499 m)	2.5	0.74	4.99	2.4×10^7–7.6×10^7	970–3095	8.4–15.2	4.4–5.1

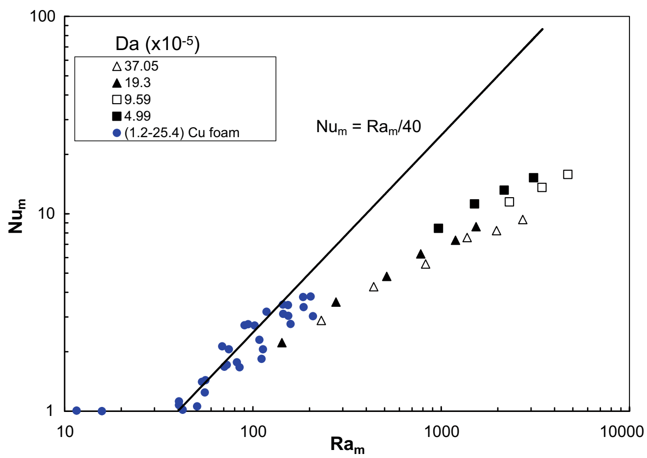

Fig. 2. Nusselt number versus porous medium Rayleigh number for RVC and copper foam [1]. Open symbols correspond to 10 PPI, and closed, to 20 PPI foam for water–RVC. The solid line is the Elder relation [7].

of solid conductance to overall heat transfer through the foam–water combination with buoyant flow. However, by considering foam materials as a structural class, heat transfer via conduction through the matrix cannot generally be neglected. A correlation that consolidates the copper and RVC foam data is achieved by introducing the conductivity ratio k_m/k_f, which varies by an order of magnitude for the two foam–water systems. The correlation,

$$Nu_m = (0.008 \pm 0.003)\left(\frac{k_m}{k_f}\right)^{(0.25 \pm 0.04)} Ra_m^{0.50 \pm 0.02} Pr_p^{0.38 \pm 0.04}, \quad (3)$$

consolidates the present data with that of Kathare et al. [1] where the range of parameters is the same as in Eq. (2), and $R^2 = 0.995$ (Fig. 3).

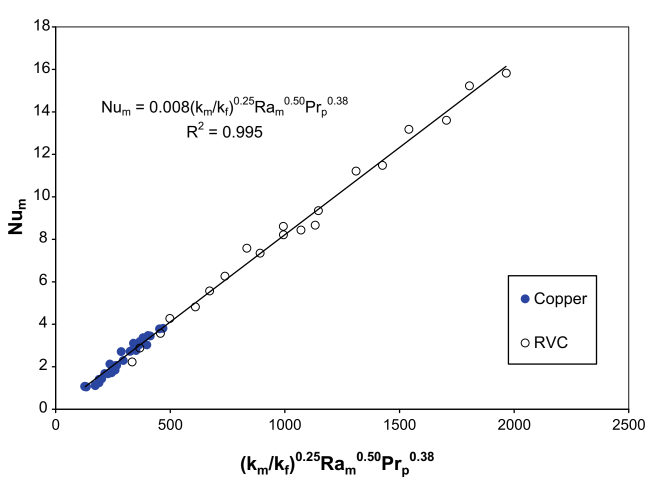

Fig. 3. Heat transfer correlation for copper and RVC foam–water layers.

3.2. Heat transfer enhancement

The utility of high porosity foam in heat transfer applications is its potential to enhance heat transfer rate above that which would be achieved in the fluid alone. Measured Nusselt numbers versus Rayleigh number are plotted in Fig. 4 in terms of the fluid properties. Included are the data of Kathare et al. [1] for water-saturated copper foam and the Garon–Goldstein correlation [10]. The presence of the water-saturated copper foam generally produces larger fluid Nusselt numbers than for water, but the presence of the RVC foam decreases the fluid Nusselt number below the water value. Thus, while the carbon foam produces drag effects similar to those observed for the copper foam, it lacks sufficient material conductance for overall heat transfer enhancement.

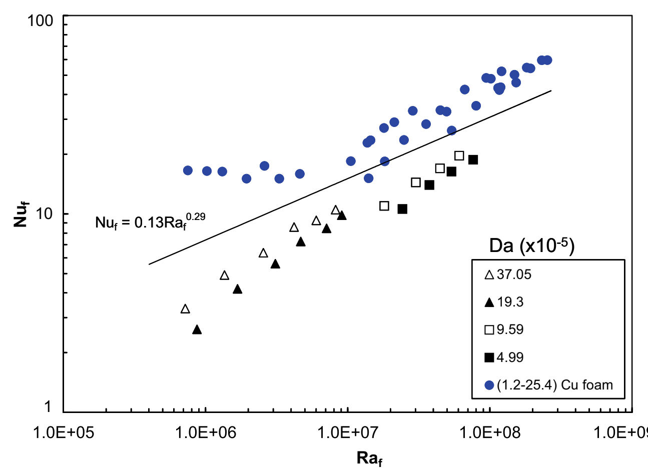

Fig. 4. Heat transfer data in terms of fluid Nusselt and Rayleigh numbers. Copper foam–water data is due to Kathare et al. [1] and the solid line is for free convection in a water layer [10]. Open symbols correspond to 10 PPI, and closed, to 20 PPI foam for water–RVC.

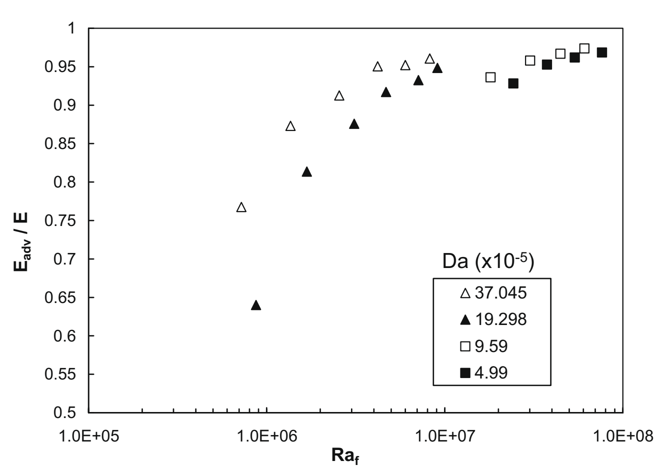

Fig. 5. Ratio of advective enhancement to overall enhancement.

Table 3
Hydrodynamic and heat transfer parameters.

$Da \times 10^5$	37.05	19.30	9.59	4.99
Ra_f	7.21×10^5–8.23×10^6	8.72×10^5–9.09×10^6	1.81×10^7–6.09×10^7	2.43×10^7–7.64×10^7
L (m)	0.0254	0.0254	0.0499	0.0499
PPI	10	20	10	20
$d_p \times 10^3$ (m)	~2.54	~1.27	~2.54	~1.27
$\delta \times 10^3$ (m)	1.4–4.4	2.2–8.6	1.6–2.9	1.6–2.9
Pr_m	4.9–6.2	4.7–5.9	4.7–5.5	4.4–5.1
u_m (m/s)	0.0014–0.011	0.0009–0.008	0.0042–0.011	0.0032–0.0086
k_d (W/m-K)	0.07–0.57	0.03–0.29	0.21–0.56	0.12–0.32
k_d/k_m	0.10–0.84	0.043–0.42	0.28–0.75	0.16–0.43
Form drag/Darcy drag	0.06–0.55	0.02–0.21	0.17–0.52	0.09–0.29
Brinkman drag/Darcy drag	0.0003–0.0053	0.0001–0.0017	0.0009–0.0043	0.0006–0.0024

For a given Darcy number, when the fluid Rayleigh number is increased, the reduction in Nusselt number from a water layer decreases. Nevertheless, for all Rayleigh numbers the RVC foam reduces the Nusselt number, and the enhancement factor, E, defined as the ratio of Nusselt number with the foam present to Nusselt number of water alone, is less than unity, or

$$E = \left.\frac{Nu_{\mathrm{f,\,foam}}}{0.13Ra_{\mathrm{f,water}}^{0.29}}\right|_{Ra_f} < 1. \tag{4}$$

However, the importance of advection to the overall heat transfer rate can be seen by comparing the enhancement factor, E, to the advective enhancement factor [1],

$$E_{\mathrm{adv}} = \left.\frac{(Nu_m - 1)}{(0.13Ra_f^{0.29} - 1)}\left(\frac{k_m}{k_f}\right)\right|_{Ra_f}. \tag{5}$$

In Fig. 5, $E_{\mathrm{adv}}/E < 1$ for the present range of Ra_f, but it increases with Ra_f to account for more than 90% of the heat transfer rate for $Ra_f > 10^7$.

4. Discussion

To explore the causes of the deviation from the packed bed relation, parameters affecting natural convection in porous media are estimated and compared (Table 3) following the scale analysis presented by Kathare et al. [1]. These parameters are calculated using a nominal pore diameter estimated from the definition PPI of the foam and the mean temperature across the foam layer. It is to be noted that pore diameter given in Table 3 is estimated as the maximum value as was the case in Ref. [1]. For the present study, $d_p = 0.0254/(d_l + d_p) < 0.0254/d_p$. Using a volume averaged value based on relative density and hence average porosity, uncertainty in pore diameter is at most 17%, which does not significantly affect our assessment of several competing factors in the over transport process.

The thermal boundary layer thickness is approximated by Elder's approximation for packed beds, i.e., $\delta \sim L/2Nu_m$ [7], and the velocity scale is estimated from a balance of drag and buoyancy [1]. Estimated values of δ are approximately the same as the pore diameter (Table 3). For the copper foam, Kathare et al. [1] report a thermal boundary layer thickness, based on a similar analysis, approximately an order of magnitude larger than the pore diameter for $Ra_m < 200$.

The ratio of form drag to Darcy drag is estimated by,

$$\frac{\text{From drag}}{\text{Darcy drag}} \sim \frac{\rho_f C_F u_m^2/\sqrt{K}}{\mu_f u_m/K}. \tag{6}$$

Form drag varies greatly over our data but can reach ~52–55% of Darcy drag in the 10 PPI foam, signifying a significant influence of inertia at high Rayleigh number. Brinkman drag is compared to Darcy drag by the ratio,

$$\frac{\text{Brinkman drag}}{\text{Darcy drag}} \sim \frac{\mu_f u_m/(Pr_m\delta)^2}{\mu_f u_m/K} = \frac{K}{(Pr_m\delta)^2} \tag{7}$$

Brinkman drag reaches a maximum of ~0.5% of Darcy drag, and thus based on our estimate of boundary layer thickness does not appear to contribute significantly to momentum considerations. Brinkman effects on heat transfer are assumed to be insignificant for the present data.

Dispersion, the pore scale mixing caused by flow separation around ligaments, can be quantified as equivalent dispersion conductivity. Dispersion conductivity in metal foams is most commonly modeled by [12]

$$k_d = 0.025\rho_f C_F u_m \sqrt{K}. \tag{8}$$

Dispersion conductivity increases with velocity and is highest in the same cases where inertial drag is large. For the present data maximum dispersion conductivity reaches 84% of stagnant thermal conductivity.

Collectively these estimates indicate that the growth of form drag caused by the significant difference in geometry between foam and packed beds plays a major role in the observed relation between Nu and Ra_m. However, for the RVC foam–water system, conductivity effects dominate, and the introduction of the RVC foam into the water layer reduces overall heat transfer rates. A similar result has been reported [3] for different foam geometry.

5. Conclusion

This study extends the work of Kathare et al. [1] and further characterizes the role of solid:fluid conductivity ratio on free convection in open cell foam structures in which a transition to adjectively dominated transport occurs. Water-saturated RVC foam reduces overall heat transfer for $148 \leqslant Ra_m \leqslant 4700$ in 10 and 20 PPI. The present data have enabled development of more general correlation (Eq. (3)) for mean Nusselt number by inclusion of the conductivity ratio, k_m/k_f.

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